15. a) Derive an expression for the maximum strain energy stored in the closed coil spring under axial load in term of maximum shear stress, modulus of rigidity and volume of the spring.

b) An automobile helical coil spring is to have a mean diameter of 80m and stiffness 200N/mm. The total axial force is 8000 N and allowable shear stress of spring material is 320N/mm$^2$ and G = 8 x 10$^4$ N/mm$^2$. Calculate.

i) Diameter of coil, ii) Number of effective coils, iii) Free length of spring and iv) Maximum energy which can be stored in the spring.

c) A laminated semielliptical leaf spring under central load of 10kN is to have an effective length of 1m and not to deflect more than 75mm. The spring has 10 leaves, 2 of which are full length and have been pre-stressed so that all leaves have same stress at full load condition. All the leaves have same width and thickness. The maximum allowable stress in the leaves is 350N/mm$^2$. Calculate the width and thickness of leaves.

**VTU, Dec. 08/Jan. 09**

# UNIT 4

# DESIGN OF SPUR AND HELICAL GEARS

## 4.1 INTRODUCTION

Gears are used to transmit motion or power from one shaft to another preferably if the centre distance between the two shafts is small. It is a positive and smooth drive. It is possible to drive shafts that are parallel, intersecting, or neither parallel nor intersecting by the use of gears. The most commonly used types are (i) spur gear, (ii) Helical gear (iii) Bevel gear (iv) worm and worm wheel (v) Spiral gear (vi) Rack and pinion.

## 4.2 CLASSIFICATION

Gears are classified as follows.

i) According to the relative position of axes, gears are classified as parallel axes, intersecting axes and non intersecting and nonparallel axes gears.

   If the axes of the shaft on which the gears are mounted are parallel, it is called parallel axes gears.

   Example : Spur gear, Helical gear, Double Helical gear.

   If the axes of the shaft on which the gears are mounted are intersecting, it is called intersecting axes gears.

   Example : Bevel gear [straight, spiral or zerol]

   If the axes of the shaft on which the gears are mounted are neither parallel nor intersecting, it is called non parallel and non intersecting axes gears.

   Example : Crossed Helical gear, worm gear, Hypoid gears

ii) According to the peripheral velocity of gears, it is classified as low velocity, medium velocity and high velocity gears.

   If the speed is less than 3 m/sec, it is termed as low velocity gears. Similarly if the speed is between 3 to 15 m/sec it is termed as medium velocity gears and if it is more than 15 m/sec, it is called high velocity gear.

iii) According to the type of gearing, it is classified as internal gearing and external gearing.

In internal gearing, the gears mesh internally and they rotate in same direction. In external gearing, the gears mesh externally and they rotate in opposite direction.

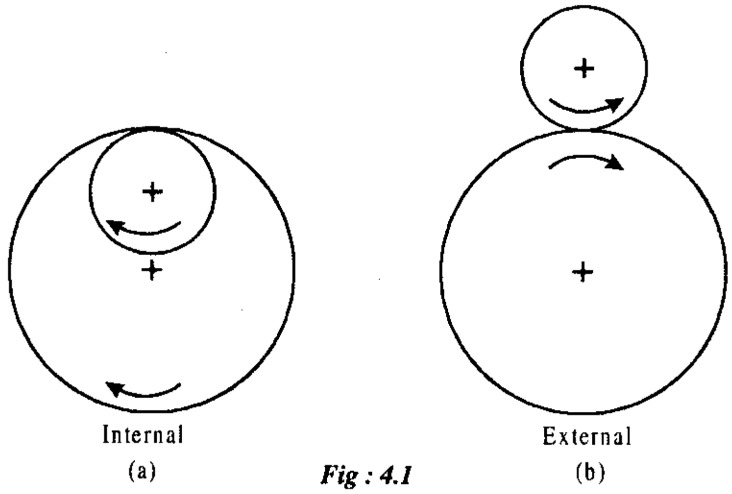



*Fig : 4.1*

iv) According to the position of teeth on the gear surface, it is classified as straight, inclined and curved teeth gears.

Straight teeth - Ex : spur gear.

Inclined teeth - Ex : Helical gear

Curved teeth - Ex : Spiral gear.

## 4.3 SPUR GEAR

Gears whose axes are parallel and whose teeth are parallel to the centre line of the gear are called spur gears. It is used to transmit power between two parallel shafts. It is widely used right from small watches to gear boxes fitted in aeroplanes.

### 4.3.1 Spur Gear Terminology

i) **Circular Pitch :** It is the distance measured along the circumference of the pitch circle from a point on one tooth to a corresponding point on the adjacent tooth. It is denoted by $p_c$.

$$p_c = \pi m \qquad \text{where } m = \text{module.}$$

ii) **Diametral Pitch :** It is expressed as the number of teeth per unit length on the pitch circle diameter. It is denoted by $p_d$.

$$\therefore \quad p_d = \frac{z}{d} \quad \text{where } z = \text{number of teeth}, \ d = \text{pitch circle diameter.}$$

iii) **Module :** It is the length of pitch circle diameter per tooth. It is denoted by 'm' $\therefore \ m = \frac{d}{z}$

iv) **Black lash :** It is the difference between the thickness of a tooth and the width of the tooth space in which it meshes.

v) **Clearance :** It is the difference between the addendum of one gear and the dedendum of the mating gear.

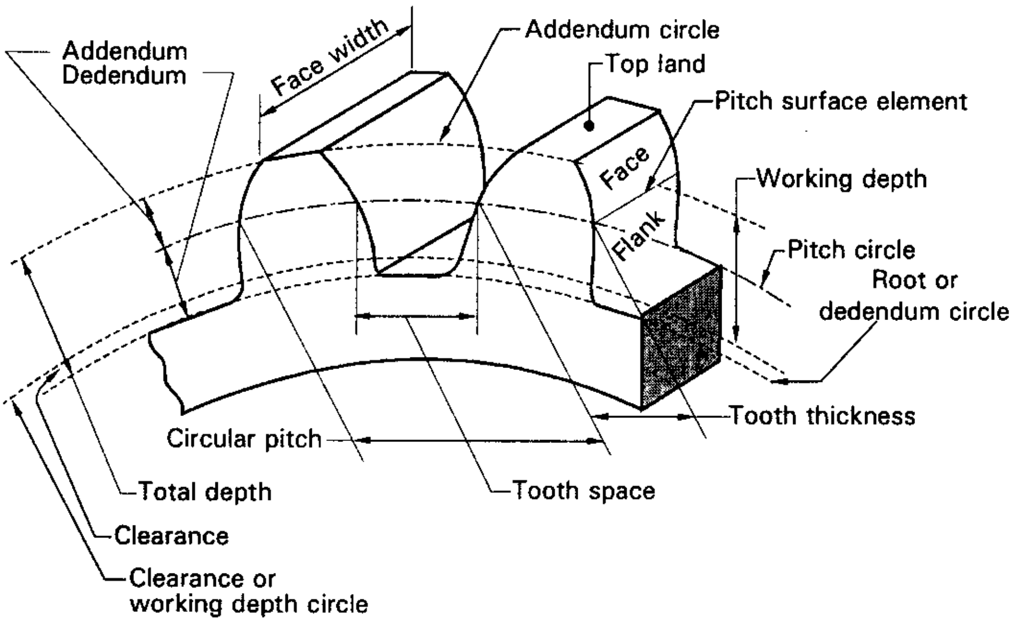



*Fig : 4.2*

vi) **Pitch point :** The point of contact of two mating gears pitch circle is known as pitch point.

vii) **Angle of obliquity or Pressure angle :** It is the angle between the common normal at the point of contact of two teeth and the common tangent to the pitch point and is denoted by $\phi$.

viii) **Path of contact :** It is the path traced by the contact point of a pair of tooth profiles from the beginning of engagement to the end of engagement.

ix) **Path of Approach :** The portion of path of contact from the beginning of engagement to the pitch point is known as path of approach.

x) **Path of recess :** The portion from the pitch point to the end of engagement is termed as path of recess.

xi) **Arc of contact :** It is the locus of a point on the pitch circle from the beginning of engagement to the end of engagement of a pair of teeth in mesh. The part from the beginning of engagement to the pitch point is termed the arc of approach and that from the pitch point to the end of engagement is termed the arc of recess. The sum of these two is the arc of contact.

xii) **Contact Ratio :** It is the ratio of angle of action to pitch angle and also the ratio of the length of line of action to the base pitch. It is a number which indicates the average number of pairs of teeth in contact.

$$\therefore \quad \text{Contact Ratio} = \frac{\text{Path of contact}}{\text{Base pitch}} = \frac{\text{Path of contact}}{\pi\, m \cos\phi} = \frac{\text{Arc of contact}}{p_c}$$

xiii) **Pitch angle :** It is the angle subtended by an arc on the pitch circle equal in length to the circular pitch.

xiv) **Base pitch :** It is the distance measured along the base circle from a point on one tooth to the corresponding point on the adjacent tooth and is denoted by $p_b$.

$$\therefore \quad p_b = \pi m \cos \phi = p_c \cos \phi \text{ where } \phi \text{ is pressure angle}$$

xv) **Addendum :** It is the radial height of tooth above pitch circle.

xvi) **Dedendum :** It is the radial depth of tooth below pitch circle.

xvii) **Addendum circle :** The circle which passes through the tips of all tooth is known as addendum circle.

xviii) **Dedendum circle :** The circle which passes through the root of all tooth is known as root or dedendum circle.

xix) **Pitch circle :** An imaginary circle passing through the pitch point having its centre at the axis of the gear.

xx) **Face :** The surface above the pitch surface is called the face.

xxi) **Flank :** The surface below the pitch surface is called the flank.

## 4.4 LAW OF GEARING OR CONDITION FOR CORRECT GEARING

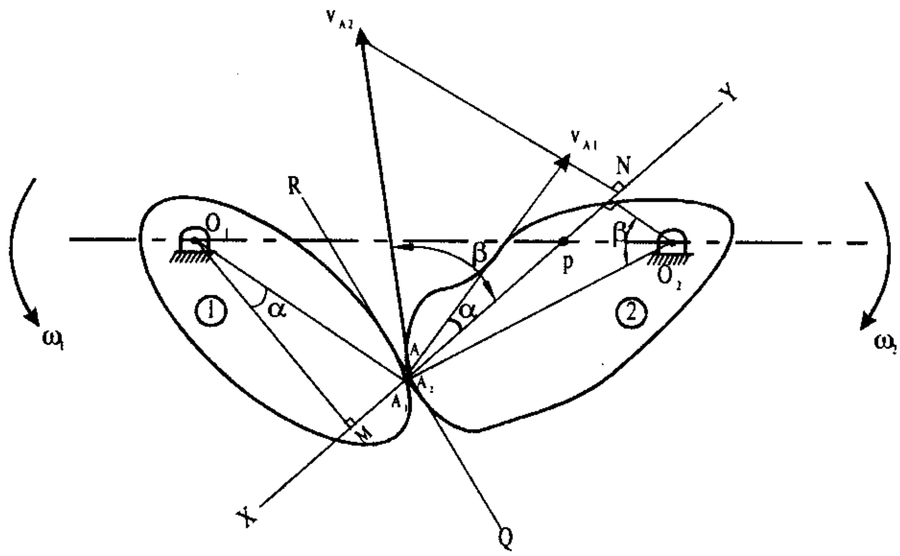



*Fig : 4.3*

When the tooth profiles are so shaped, and if they produce a constant angular velocity ratio during meshing, then they are said to have conjugate action. Involute tooth profile is one of them which gives conjugate action.

Let two curved bodies 1 and 2 rotating about centres $O_1$ and $O_2$ be in contact at A as shown in Fig. 4.3.

$A_1$ and $A_2$ are two coincident points. $A_1$ lying on body 1 and $A_2$ lying on body 2. RQ and XY are common tangent and common normal at the point of contact respectively. $\omega_1$ and $\omega_2$ are the angular velocities of $A_1$ and $A_2$ respectively. The linear velocities at the point of contact at the instant be $v_{A1}$ and $v_{A2}$. The direction of these are perpendicular to the line joining $O_1$ to $A_1$ and $O_2$ to $A_2$ respectively.

Let the common normal intersect the line joining the centres of rotation 1 and 2 be at P. Let $O_2N$ be a perpendicular to the common normal from $O_2$ and $O_1$ M be a perpendicular to the common normal from $O_1$. If the two bodies are to remain in contact, then the component of linear velocities of $A_1$ and $A_2$ along the common normal must be equal.

i.e., $$v_{A1} \cdot \cos\alpha = v_{A2} \cdot \cos\beta$$

i.e., $$\omega_1.O_1A_1 \cos\alpha = \omega_2.O_2A_2 \cdot \cos\beta$$

$$\therefore \quad \omega_1 .\ O_1A_1 \frac{O_1M}{O_1A_1} = \omega_2.O_2A_2 .\frac{O_2N}{O_2A_2}$$

$$\therefore \quad \frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \text{Velocity ratio}$$

Also triangles $O_1$ MP and $O_2$ NP are similar.

$$\therefore \quad \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P}$$

$$\therefore \quad \text{Velocity ratio} = \frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P}$$

Thus for constant angular velocity ratio of gearing, the normal at the point of contact divides the line joining the centres of rotation in the inverse ratio of the angular velocities.

Therefore the contact surfaces of the gear teeth should be so shaped that the common normal at the point of the mating teeth cuts the line joining the centres of rotation of gear wheels at P and P is the pitch point.

Thus the law of gearing states that for constant angular velocity ratio of the two gears, the common normal at the point of contact of the two meshing teeth must pass through a fixed point on the line joining the centres of rotation. The fixed point is the pitch point in the case of gears.

## 4.5 INTERFERENCE AND METHODS TO AVOID INTERFERENCE

Mating of two non-conjugate teeth is known as interference.

Gear teeth are said to interference when they have to overlap or cut into the mating teeth.

In Fig. 4.4 the addendum of a rack has been chosen so that the first point of contact will occur at point E, which is the point of tangency for the line of action and the base circle of the pinion. The involute profile on the pinion cannot extend inside the base circle. From the base circle inward the pinion profile is drawn as a radial line. Then the maximum length of the line of approach is EP. The maximum addendum which should be used on the rack is *a*. In order to investigate that happens if the addendum of the rack were larger, it is shown as a'. Then if the pitch circle of the pinion and the pitch line of the rack were rolled to the right, the rack and pinion-tooth positions would be as shown by the dotted profiles, and it is found that the rack tooth overlaps or interferes with the pinion tooth. In order to satisfy the fundamental law of gearing, it is necessary to undercut the pinion teeth as shown at the right. This weakens the teeth and removes a part of the involute profile, which shortens the path of contact.

There can be no interference due to a large addendum on the pinion since the point of tangency of the line of action with the base circle for the rack lies to the left at infinity. The maximum length for the path of recess is PB' and results when the addendum of the pinion is increased to the extent that the pinion tooth becomes pointed.

If a rack will mesh with a pinion without interference, then any finite external gear having the same addendum as the rack will mesh with the pinion without interference. This can be seen from Figure 4.4. The addendum circle of any finite gear will intersect the line of action to the left of E.

In order to avoid interference the following method may be employed.

i) Height of the teeth may be reduced.

ii) The radial flank of the pinion may be cut back, known as under cutting.

iii) Centre distance may be increased. It leads to increase in pressure angle.

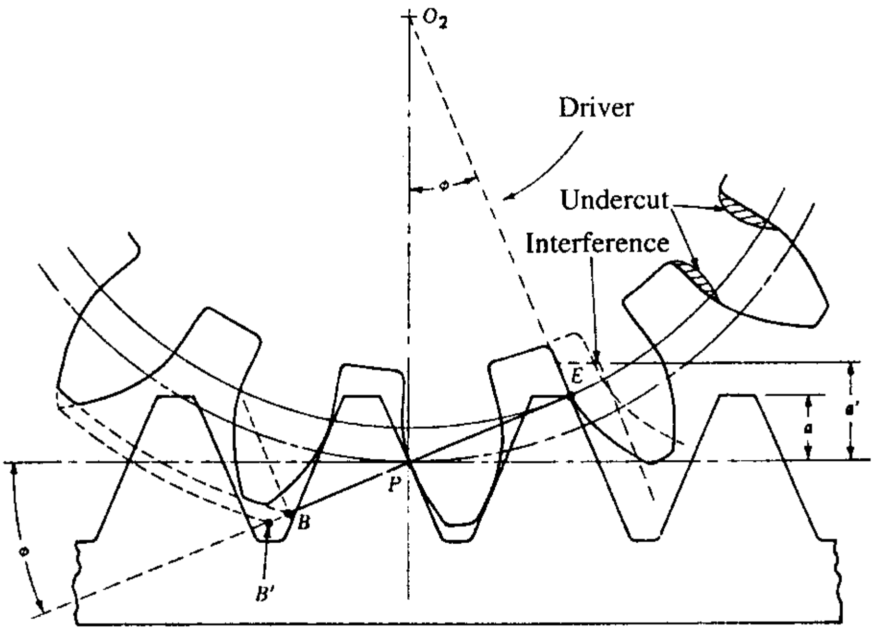



*Fig. 4.4*

iv) By tooth correction. (The face of gear teeth may be relieved) i.e., the pressure angle, centre distance and base circles remain unchanged but tooth thickness of gear becomes greater than p/2 and pinion becomes less than p/2.

## 4.6 PROFILE

The curve forming face and flank is called profile. Basically two types of profiles are used in gears. One is involute and the other cycloid. If the curve is of involute in nature then the teeth is called involute teeth and if it is cycloidal in shape, then the teeth is called cycloidal teeth.

### 4.6.1 Advantages of Involute Profile

1. In involute profile pressure angle is constant throughout the engagement i.e.., from the point of commencement to point of disengagement, where as in cycloidal profile the pressure angle is zero at pitch point and is maximum at the point of engagement and at the point of disengagement.
2. In involute profile, a small variation in centre distance does not affect the velocity ratio where as in cycloidal profile the centre distance should be maintained very accurately.
3. In involute profile the curve is a single curve, so it is easy to manufacture, where as in cycloidal profile the curve comprises of two namely epicycloid and hypo-cycloid, so it is difficult to manufacture.

### 4.6.2 Advantage of cycloidal profile :

1. In cycloidal profile there is no interference whereas in involute profile interference will occur.
2. Cycloidal profile have spreading flanks whereas involute profile have radial flanks. So cycloidal tooth is stronger than involute tooth.
3. In cycloidal profile always one concave surface will be in contact with one convex surface so the wear and tear will be minimum when compared with involute profile.

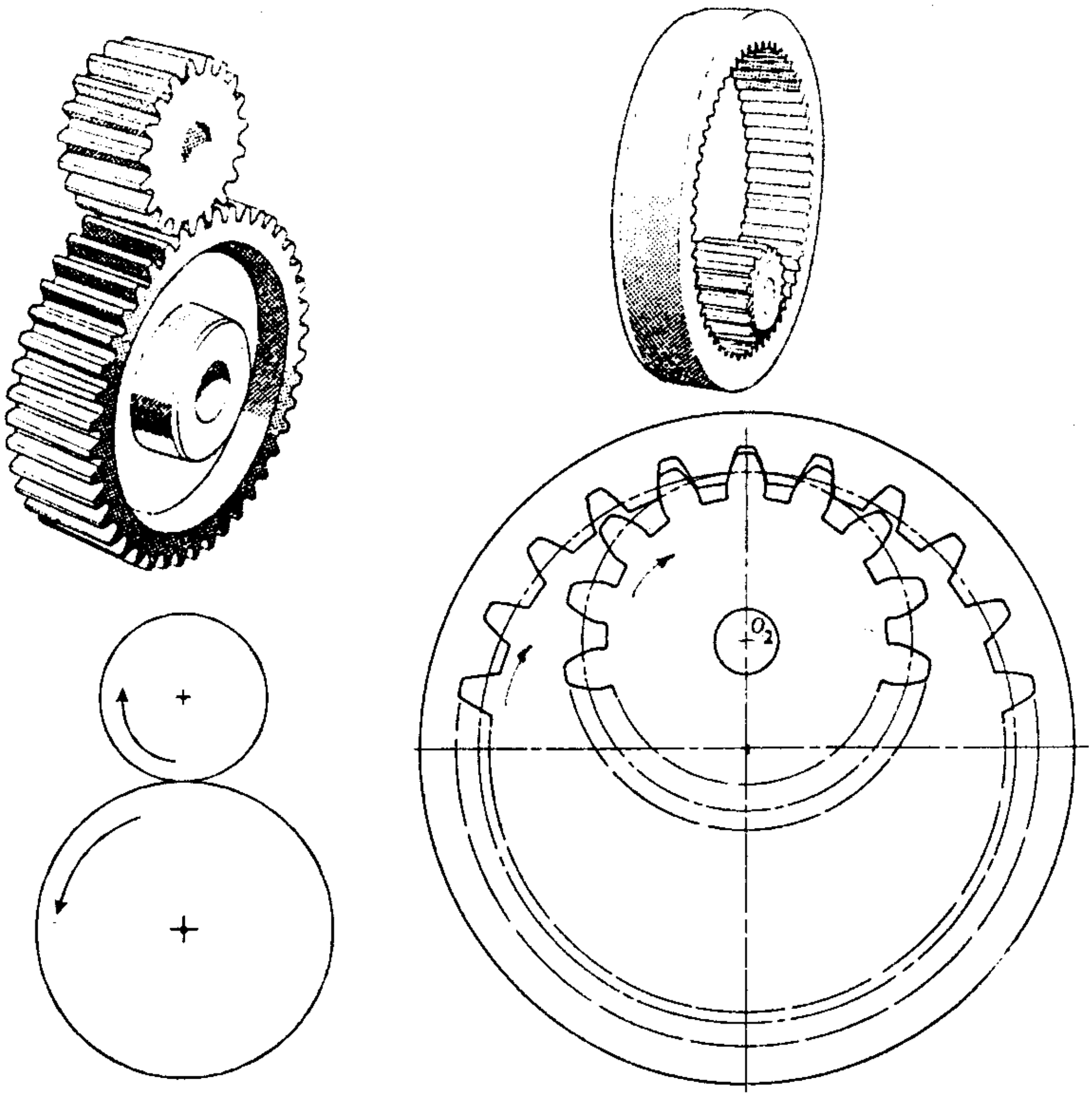
### 4.6.3 Properties of involute tooth profile

1. A normal to an involute is a tangent to the base circle.
2. When involutes are in mesh, then the pressure angle remain constant.
3. The involute is the only tooth form which is insensitive to the centre distance of its base circle.
4. The shape of the involute profile depends only on the dimension of the base circle.
5. The radius of curvature of an involute is equal to the length of tangent to the base circle.
6. Basic rack for involute tooth profile has straight line form.
7. The common tangent to the base circles of the two involutes is the line of action and also the path of contact between the involutes.
8. When two involutes are in mesh, then they transmit constant angular velocity ratio and it is inversely proportional to the size of base circles.
9. Manufacturing is easy due to single curvature.
10. Involute gears in mesh give conjugate action.
11. Suitable for motion and power transmission.

## 4.7 INTERNAL AND EXTERNAL SPUR GEAR

Fig. 4.5 (a) shows the spur gears with external teeth on the outer surface of the cylinders and hence the shafts rotate in the opposite directions.

Fig. 4.5 (b) shows the spur gear with internal teeth. In an internal spur gear, the teeth are formed on the inner surface of the annulus ring. An internal gear can mesh with external pinion only and the two shafts rotate in the same direction. Internal gear will have greater length of contact, greater tooth strength and lower relative sliding between meshing teeth than external gear. In an internal gear the tooth profiles are concave instead of convex.




*(a) External spur gear* *(b) Internal gear*

*Fig : 4.5*

## 4.8 SYSTEMS OF GEAR TEETH

To cut the gear teeth mainly three systems are used. They are 14½° involute system, 20° full depth involute system and 20° stub involute system.

i) **14½ involute system :** Number of teeth to avoid interference is high because of low pressure angle. These profiles are obtained by the gear generating process.

ii) **20° full depth involute system :** Because of higher pressure angle, the number of teeth to avoid interference is less. These teeth are broader at the root and stronger.

iii) **20° stub involute system :** In this system the working depth is usually 20% less than the full depth system and the addendum is made shorter. The advantages of this system are:
   (i) Low production cost
   (ii) Stronger than full depth tooth
   (iii) Less interference due to shorter addendum.

However with larger number of teeth, full depth or 14½° involute system perform better than the sub involute system.

## 4.9 BEAM STRENGTH OF SPUR GEAR TEETH OR LEWIS EQUATION

The beam strength of gear teeth is determined from an equation known as Lewis equation. The load carrying ability of the gears as determined by this equation gives satisfactory results. Lewis assumed that the load is being transmitted from one gear to another, it is all given and taken by one tooth. When contact begins, the load is assumed to be at the end of the driven teeth and as contact ceases, it is at the end of driving teeth.

Consider each tooth as a cantilever beam loaded by a normal load $F_N$ as shown in Fig. 4.6. It is resolved into two components (i) tangential component ($F_t$) (ii) Radial component ($F_r$)

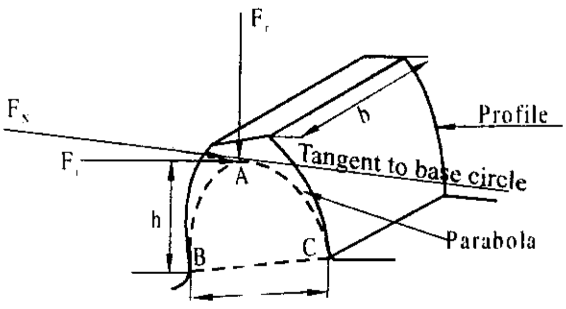



*Fig : 4.6*

The tangential component $F_t$ induces a bending stress which tends to break the tooth. The radial component $F_r$ induces compressive stress of relatively small magnitude therefore its effect on the tooth may be neglected. Hence the bending stress is used as the basis for design calculations.

The critical section may be obtained by drawing a parabola through A and tangential to the curves at B and C. This parabola shown in dotted line outlines a beam of uniform strength. But since the tooth is larger than the parabola at every section except BC, it concludes BC is the critical section.

The maximum value of bending stress at BC is given by, $\sigma = \frac{M}{I} \cdot c$ where

$$M = \text{Maximum bending moment at section BC} = F_t \times h$$

$$c = \frac{t}{2},\ t = \text{thickness of tooth}$$

$$I = \frac{bt^3}{12},\ b = \text{face width of tooth}$$

$$\therefore\ \sigma = \frac{(F_t \times h)\left(\frac{t}{2}\right)}{\left(\frac{bt^3}{12}\right)} = \frac{(F_t.h)6}{bt^2}$$

$$\therefore\ F_t = \sigma \times b \times \frac{t^2}{6h}$$

Since t and h are variables which depends upon the shape of tooth and circular pitch, the quantity $\frac{t^2}{6h}$ may be replaced by 'p y' where y is known as Lewis form factor or tooth form factor. Therefore the final form of strength formula is $F_t = \sigma bpy$. This equation is called Lewis equation or Beam strength of tooth. The value of 'y' for various tooth systems are

$y = 0.124 - \frac{0.684}{z}$ for 14½° involute ---- 2.97(Old DDHB); **23.115 (New DDHB)**

$= 0.154 - \frac{0.912}{z}$ for 20° F D involute---- 2.98(Old DDHB); **23.116 (New DDHB)**

$= 0.170 - \frac{0.95}{z}$ for 20° stub teeth ---- 2.99(Old DDHB); **23.117 (New DDHB)**

The permissible stress in Lewis equation depends upon materials, pitch line velocity and load condition. According to Barth formula the permissible stress

$\sigma = \sigma_o \times C_v$ where $C_v$ = velocity factor and

$\sigma_o$ = allowable static stress from Table 2.16 (Old DDHB); **23.18 (New DDHB)**

$C_v = \frac{3}{3 + v_m}$ for $v_m < 7.5$ m/sec ---- 2.128 (Old DDHB); **23.134a (New DDHB)**

$= \frac{4.5}{4.5 + v_m}$ for $v_m$ upto 12.5 m/sec---- 2.129(Old DDHB); **23.135a (New DDHB)**

$$v_m = \text{Mean pitch line velocity} = \frac{\pi d_1 n_1}{60000} \text{ or } \frac{\pi d_2 n_2}{60000}$$

Therefore the final form of Lewis equation for tangential tooth load in terms of circular pitch

$$F_t = \sigma_o \, byp \, C_v = \sigma_o \, byp \, K_v$$ ---- 2.93(Old DDHB); **23.93 (New DDHB)**

## 4.10 DYNAMIC TOOTH LOAD

The dynamic loading is due to the following reasons.

i) Errors in tooth spacing.

ii) The elements of the face are not perfectly parallel to the axis.

iii) Inaccuracies of the tooth profile.

iv) The load is never distributed uniformly across the face.

v) Deflection of teeth under load.

vi) Deflection of shaft and mountings.

Due to the above inaccuracies, there will be dynamic load due to shock and impact. The dynamic load will be greater than the steady load. It consists of the tangential tooth load $F_t$ required for power transmission and an increment load $F_i$ caused by irregularities.

i.e., Dynamic load $F_d = F_t + F_i$

According to Buckingham's equation

$$F_d = F_t + \frac{21v(F_t + bC)}{21v + \sqrt{F_t + bC}}$$ ---- 2.148 a(Old DDHB); **23.155 (New DDHB)**

where $F_t$ = Tangential tooth load

v = Pitch line velocity in m/sec

b = Face width

C = Coefficient that depends upon the materials, pressure angle and error in action.

If class of gear is known then error 'f' is obtained from Fig. 2.29(Old DDHB); Fig. 23.34a (New DDHB) otherwise based on velocity v, the error f is calculated from Table 2.34 (Old DDHB); Fig. 23.35a (New DDHB). Now the value C is estimated from Table 2.35 (Old DDHB); Table 23.32 (New DDHB). The ratio of dynamic load and tangential transmitted load is called dynamic load factor. It is a function of peripheral velocity, tooth surface hardness and degree of accuracy. If the gear tooth is not strong enough statically or dynamically then

i) The module can be increased.

ii) The face width can be increased.

iii) The gear material or hardness can be changed.

## 4.11 DESIGN FOR WEAR

There are two main reasons for gear tooth failures (i) Breakage of the tooth due to static and dynamic loads (ii) Surface destruction. The complete breakage of the tooth can be avoided by changing the parameters such as module or face width. Due to rolling and sliding action of the gear teeth, the following types of surface destruction or tooth wear may occur:

i) **Abrasive wear :** A failure due to the presence of foreign material in the lubricant that can scratch the tooth surface.

ii) **Corrosive wear :** A failure due to chemical reaction on the tooth surface.

iii) **Pitting :** A fatigue failure due to repeated application of stress cycles.

iv) **Scoring :** A failure due to metal to metal contact due to the break down of the oil film.

The failure of the gear tooth due to pitting occurs when the contact stresses between two meshing teeth exceed the surface endurance strength of the material. Pitting is a surface fatigue failure. Buckingham analysed the wear strength and gave an equation for gear tooth. According to Buckingham's equation, wear strength of gear tooth is $F_w = d_1 bQK$ ---- 2.160(Old DDHB); **23.160 (New DDHB)**

where $d_1$ = Pitch diameter of pinion in meter

$b$ = Face width of gear in meter

$Q$ = Ratio factor $= \dfrac{2z_2}{z_1 + z_2} = \dfrac{2d_2}{d_1 + d_2}$ ---- 2.163(Old DDHB); **23.163 (New DDHB)**

$z_1$ = Number of teeth on pinion; $z_2$ = Number of teeth of gear

$K$ = Load stress factor $= \dfrac{1.43\sigma_{-1c}^2 \sin\alpha}{E_o} = \dfrac{1.43\sigma_{fac}^2 \sin\alpha}{E_o}$ ---- 2.161(Old DDHB); **23.161 (New DDHB)**

$E_o$ = Equivalent Young's modulus $= \dfrac{2E_1E_2}{E_1 + E_2}$ ---- 2.162(Old DDHB); **23.162 (New DDHB)**

$E_1$ = Young's Modulus of pinion in N/m$^2$

$E_2$ = Young's Modulus of gear in N/m$^2$; $\alpha$ = Pressure angle

$\sigma_{fac} = \sigma_{-1c}$ = Surface fatigue strength of gear material = $(2.75\, H_B - 69)$ MPa ---- 2.168 c(Old DDHB); **23.168 (New DDHB)**

$H_B$ = Average value of BHN for the gear pair.

The wear load $F_w$ must be greater than dynamic load $F_d$. In case $F_w$ is less than $F_d$, the pinion and gear materials are not hard enough and hence the suitable treatment should be given. The greater the ratio $\dfrac{F_w}{F_d}$, the longer the gear life.

Also load stress factor $K = 0.16\left(\dfrac{BHN}{100}\right)^2$. This relationship is suitable only when both the gears are made of steel with a pressure angle of 20°.

## 4.12 ENDURANCE STRENGTH

The endurance strength of the gear tooth is given by $F_B = F_{-1} = \sigma_{-1}\, bYm = \sigma_{sf}\, bYm$

---- 2.153(Old DDHB); 23.158 (New DDHB)

where $Y = \pi y$, $y$ = Lewis form factor

$\sigma_{sf} = \sigma_{-1}$ = endurance limit of the gear material from Table 2.39 (Old DDHB); **Table 23.33 (New DDHB)**

The dynamic load should be less than the endurance strength by a reasonable margin of safety.

## 4.13 SELECTION OF MATERIAL

The desirable properties of a gear material are:

i) The load carrying capacity of the gear tooth depends upon the ultimate tensile or yield strength of the material. Endurance strength is a deciding factor if the gear tooth is subjected to fluctuating loads.

ii) The dimensions of the gear tooth is decided by wear rating rather than strength rating in many cases.

iii) For high speed to avoid failure due to scoring the material should have a low coefficient of friction.

iv) To minimise the amount of thermal distortion or wrapping during heat treatment process, alloy steels are preferred rather than plain carbon steel.

Gears are usually made of cast iron, steel, bronze or phenolic resins. Large size gears are made of grey cast iron.

## 4.14 DESIGN CONSIDERATIONS

For the design of a gear drive, the following datas are usually given:

i) Power to be transmitted.

ii) Speed of pinion or gear (i.e., driver or driven).

iii) Centre distance.

While attempting the design of a gear drive, the following points should be taken into consideration:

i) The teeth must be sufficiently strong so as to withstand the static loading as well as dynamic loading on the drive.

ii) The teeth should have good wear resisting properties for their long life.

iii) The drive should be compact.

iv) The drive should be properly aligned.

v) The gears must operate without interference.

vi) The teeth must be able to resist tooth deflection, stress concentration and accelerations.

vii) Proper lubrication arrangement should be made.

## Procedural steps for the design of spur gear

Let $m$ = Module

$z_1$ = Number of teeth on pinion

$z_2$ = Number of teeth on gear

$n_1$ = Speed of pinion in rpm

$n_2$ = Speed of gear in rpm

$i$ = Velocity ratio

$\alpha$ = Pressure angle

$P = N$ = Power in kW

$F_t$ = Tangential tooth load in N

$F_d$ = Dynamic load in N

$F_w$ = Wear load in N

$F_f = F_{-1}$ = Endurance strength in N

$\sigma_{01}$ = Allowable static stress of pinion in N/mm$^2$

$\sigma_{02}$ = Allowable static stress of gear in N/mm$^2$

$E_1$ = Young's Modulus of pinion in N/mm$^2$

$E_2$ = Young's Modulus of gear in N/mm$^2$

$E_o$ = Equivalent Young's Modulus in N/mm$^2$

$y$ = Lewis form factor

$C_s$ = Service factor ---- Table 2.33 (Old DDHB); Table 23.13 (New DDHB)

$K_v$ or $C_v$ = Velocity factor (Page 23.76)

$a$ = Centre distance in mm

$b$ = Face width in mm

$p$ = Circular pitch in mm

$K$ = Load stress factor

$Q$ = Ratio factor

$v_m$ = Pitch line velocity in m/sec.

$h_a$ = Addendum

$h_f$ = Dedendum

$da_1$ = Addendum circle diameter of pinion

$da_2$ = Addendum circle diameter of gear

$df_1$ = Dedendum circle diameter of pinion

$df_2$ = Dedendum circle diameter of gear

$h$ = Total depth

$c$ = Clearance

s = Tooth thickness

$\sigma_{sf}$ or $\sigma_{-1}$ = Endurance limit of gear material Table 2.39 (Old DDHB); Table 23.33 (New DDHB)

BHN = Brinell Hardness Number.

## i) Identify the weaker member

To decide the weaker member among the two, the following table has to be formulated.

| Particulars | $\sigma_o$ | y | $\sigma_o y$ | Remarks |
|---|---|---|---|---|
| Pinion | $\sigma_{o1}$ | $y_1$ | $\sigma_{o1}\ y_1$ | |
| Gear | $\sigma_{o2}$ | $y_2$ | $\sigma_{o2}\ y_2$ | |

From Table 2.16 (Old DDHB); Table 23.18a (New DDHB) obtain the allowable static stress $\sigma_{o1}$ and $\sigma_{o2}$ for the given materials.

For the given pressure angle and tooth form, Lewis form factor y for pinion and gear can be obtained by using the following formulas.

$$y = 0.124 - \frac{0.684}{z} \quad \text{for } 14\tfrac{1}{2}^\circ$$ ---- 2.97 (Old DDHB); 23.115 (New DDHB)

$$y = 0.154 - \frac{0.912}{z} \quad \text{for } 20^\circ \text{ F D}$$ ---- 2.98(Old DDHB); 23.116 (New DDHB)

$$y = 0.170 - \frac{0.95}{z} \quad \text{for } 20^\circ \text{ stub}$$ ---- 2.99(Old DDHB); 23.117 (New DDHB)

Since the load carrying capacity of the tooth is a function of the product $\sigma_o$ y, the gear whose value of $\sigma_o$ y is less is the weaker member

i.e., if $\sigma_{01}\ y_1 < \sigma_{02}\ y_2$, pinion is weaker

if $\sigma_{02}\ y_2 < \sigma_{01}\ y_1$, gear is weaker

Design should be based on weaker member.

## ii) Design

### *a) Tangential tooth load*

$$F_t = \frac{9550NC_s}{nr} = \frac{9550PC_s}{nr}$$ ---- 2.87 b(Old DDHB); 23.87b (New DDHB)

where n = Weaker member speed ; N or P = Power in kW

$$r = \text{Radius of weaker member} = \frac{d}{2} = \frac{mz}{2}$$

$C_s$ = Service factor from Table 2.33 (Old DDHB); Table 23.13 (New DDHB)

### *b) Tangential tooth load from Lewis equation*

$F_t = \sigma_o$ by p $C_v = \sigma_o$ by p $K_v$ ---- 2.93(Old DDHB); 23.93 (New DDHB)

$\sigma_o$ = Allowable static stress of the weaker member b = $3 \pi$ m to $4 \pi$ m or 9.5 m to 12.5m ----2.126(Old DDHB); 23.132 (New DDHB)

y = Lewis form factor of the weaker member

p = $\pi$ m

$v_m = \frac{\pi dn}{60000}$ where d and n are weaker member speed and diameter respectively.

$K_v$ or $C_v$ = Velocity factor

$= \frac{3}{3+v_m}$ for $v_m \le 7.5$ m/sec ---- 2.128(Old DDHB); 23.134a (New DDHB)

$= \frac{4.5}{4.5+v_m}$ upto 12.5 m/sec ---- 2.129(Old DDHB); 23.135a (New DDHB)

$= \frac{6.1}{6.1+v_m}$ upto 20 m/sec ---- 2.130(Old DDHB); 23.136a (New DDHB)

$= \frac{5.55}{5.55+\sqrt{v_m}}$ or $\frac{5.6}{5.6+\sqrt{v_m}}$ over 20 m/sec

---- 2.131(Old DDHB); 23.137a (New DDHB)

By equating the equations obtained from a and b, and by trial and error method find module 'm' select the standard module m from Table 2.3 (Old DDHB); Table 23.3 (New DDHB).

***c) Check for the stress***

Calculate the induced stress by the equation

$\sigma_{ind} = (\sigma_o C_v)_{ind} = \frac{F_t}{byp}$ ---- 2.93(Old DDHB); 23.93 (New DDHB)

Calculate the allowable stress $\sigma_{all} = (\sigma_o C_v)_{all}$

If $(\sigma_o C_v)_{ind} < (\sigma_o C_v)_{all}$ then the design is satisfactory.

## iii) Dimensions

Calculate all important geometric parameters of tooth profile by using the equations given in Table 2.1 (Old DDHB); Table 23.1 (New DDHB), i.e., Addendum $h_a$, Dedendum $h_f$, Tooth thickness 's', Total depth h, clearance c, outside diameter of gear and pinion.

Also calculate, tangential tooth load $F_t$, centre distance, face width and root or dedendum circle diameter of gear and pinion.

## iv) Checking

***i) Check for dynamic load***

According to Buckingham's equation

Dynamic load $F_d = F_t + \dfrac{21v(F_t + bC)}{21v + \sqrt{F_t + bC}}$ ---- 2.148 a(Old DDHB);

$F_t + \dfrac{21v_m(F_t + bC)}{21v_m + \sqrt{F_t + bC}}$ ---- 23.155 (New DDHB)

Using Tables 2.34 (Old DDHB) and 2.35 (Old DDHB) or Fig. 23.35a (New DDHB) and Table 23.32 (New DDHB) the load factor C can be obtained. If the class of gear is known then the error f can be obtained from Fig 2.29 (Old DDHB); Fig. 23.34a (New DDHB).

***ii) Endurance strength of the gear tooth*** $F_{-1} = \sigma_{-1}\, bY\, m. = \sigma_{sf}\, bY\, m$

***---- 2.153(Old DDHB); 23.158 (New DDHB)***

where $Y = \pi y$, y = Lewis form factor of the weaker member.

$\sigma_{-1}$ or $\sigma_{sf}$ = Endurance limit Table 23.33 (New DDHB) : Table 2.39 (Old DDHB)

For safer design $F_d$ must be less than the allowable endurance strength.

**Note :**

Allowable static stress ($\sigma_o$) used in the Lewis equation is $\frac{1}{3}$ of ultimate stress and $\sigma_{-1} = 0.5\,\sigma_u$

***iii) Check for wear load***

According to Buckingham's equation wear load $F_w = d_1 bQK$ ---- 2.160(Old DDHB); 23.160 (New DDHB)

where Ratio factor $Q = \dfrac{2z_2}{z_1 + z_2}$ ---- 2.163(Old DDHB); 23.163 (New DDHB)

Load stress factor $K = \dfrac{1.43(\sigma_{-1c})^2 \sin\alpha}{E_o}$ ---- 2.161(Old DDHB);

$= \dfrac{1.43\sigma_{fac}{}^2 \sin\alpha}{E_o}$ ---- 23.161 (New DDHB)

Equivalent Young's Modulus $E_o = \dfrac{2E_1E_2}{E_1 + E_2}$ ---- 2.162(Old DDHB); 23.162 (New DDHB)

Limiting stress for surface fatigue $\sigma_{fac}$ or $\sigma_{-1c}$ = (2.75 HB – 69) MPa

---- 2.168 c(Old DDHB); 23.168 (New DDHB)

$H_B$ = Brinell Hardness Number

For safer design $F_w$ must be greater than $F_d$

If $F_w < F_d$ then make one or more following changes

i) Calculate $F_d$ by decreasing error 'f' ii) Decrease the module 'm'

iii) Increase the face width 'b' iv) Increase the surface hardness.

### *Example 4.1*

**A pair of mating spur gears have 20° full depth of module 5 mm. The pitch diameter of smaller gear is 100 mm. If the transmission ratio is 4 : 1. Calculate (i) Number of teeth for each gear (ii) Addendum (iii) Dedendum (iv) Whole depth (v) Clearance (vi) Outside diameter (vii) Tooth thickness (viii) Working depth (ix) Circular pitch (x) Centre distance (xi) Base circle diameters (xii) Root or dedendum circle diameters.**

**Data : $\alpha = 20°$ FD; $m = 5$ mm; $d_1 = 100$ mm; $i = 4$**

***Solution :***

$$\text{Transmission ratio } i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1}$$

$\therefore$ Pitch circle diameter of gear $d_2 = id_1 = 4 \times 100 = 400$ mm

i) Number of teeth on pinion $z_1 = \frac{d_1}{m} = \frac{100}{5} = 20$

Number of teeth on gear $z_2 = \frac{d_2}{m} = \frac{400}{5} = 80$

**From Table 2.1 (Old DDHB); Table 23.1 (New DDHB) for 20° Full depth involute system**

ii) Addendum $h_a = 1\,m = 1 \times 5 = 5$ mm

iii) Dedendum $h_f = 1.25\,m = 1.25 \times 5 = 6.25$ mm

iv) Whole depth $h = 2.25\,m = 2.25 \times 5 = 11.25$ mm

v) Clearance $c = 0.25\,m = 0.25 \times 5 = 1.25$ mm

vi) Outside diameter of pinion $da_1 = (z_1 + 2)\,m = (20 + 2)\,5 = 110$ mm
Outside diameter of gear $da_2 = (z_2 + 2)\,m = (80 + 2)\,5 = 410$ mm

vii) Tooth thickness $s = \frac{\pi}{2}\,m = \frac{\pi}{2} \times 5 = 7.854$ mm

viii) Working depth $h' = 2\,m = 2 \times 5 = 10$ mm

ix) Circular pitch $p = \pi\,m = \pi \times 5 = 15.7$ mm ---- 2.2,(Old DDHB); 23.2 (New DDHB)

x) Centre distance $a = m\left(\frac{z_1 + z_2}{2}\right) = \frac{d_1 + d_2}{2} = \frac{100 + 400}{2} = 250$ mm ---- 2.19(Old); 23.19 (New)

xi) Base circle diameter of pinion $d_{b1} = d_1 \cos\alpha = 100 \cos 20 = 93.97$ mm ---- 2.9(Old); 23.9 (New)

Base circle diameter of gear $d_{b2} = d_2 \cos\alpha = 400 \cos 20 = 375.88$ mm

xii) Root diameter of pinion $d_{f1} = d_1 - 2h_f = 100 - 2 \times 6.25 = 87.5$ mm ---- 2.16(Old); 23.16 (New)

Root or dedendum circle diameter of gear $d_{f2} = d_2 - 2h_f = 400 - 2 \times 6.25 = 387.5$ mm

### *Example 4.2*

**A spur gear pinion 100 mm diameter has a torque of 200 Nm applied to it. The spur gear mesh with it is 250 mm in diameter. The pressure angle is 20°. Determine (i) Tangential force $F_t$ (ii) Radial or separating force $F_r$ (iii) Torque on the gear. Also show the forces acting on the wheels separately.**

**Data :**

**$d_1 = 100$ mm; $M_{t1} = 200$ Nm $= 200 \times 10^3$ Nmm; $d_2 = 250$ mm; $\alpha = 20°$**

***Solution : Assume service factor $C_s = 1$***

i) Tangential force $F_t = \frac{M_{t1}C_s}{r_1} = \frac{200\times10^3\times1}{\left(\frac{100}{2}\right)} = 4000$ N ----2.87 a(Old DDHB); 23.87a (New DDHB)

ii) Normal tooth load $F_n = \frac{F_t}{\cos\alpha} = \frac{4000}{\cos 20} = 4256.71$ N ---- 2.88(Old DDHB); 23.88(New DDHB)

Radial or separating load $F_r = F_n \sin\alpha = 4256.71 \times \sin 20 = 1455.88$ N---- 2.90(Old); 23.90 (New DDHB)

iii) Also tangential tooth load $F_t = \frac{M_{t1}C_s}{r_1} = \frac{M_{t2}C_s}{r_2}$

$\therefore$ Torque on the gear $M_{t2} = F_t.r_2 = 4000 \times \frac{250}{2} = 500 \times 10^3$ Nmm

Assume pinion is the driver and it rotates in clockwise direction, then the free body diagram of forces are as shown in Fig. 4.6.

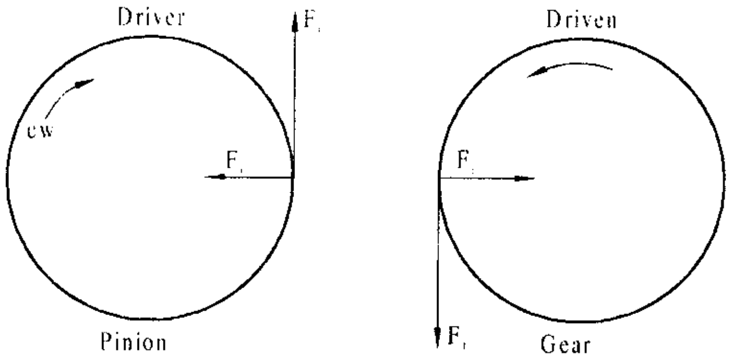



***Fig. 4.6***

***Example 4.3***

**A forged steel pinion (SAE1040) rotating at 400 rpm drives a high grade cast iron gear. The transmission ratio is 4:1. The pinion has 15 standard 20° full depth involute teeth of 4 mm module. The face width of both gear is 40 mm. How much power can be transmitted from the stand point of strength.**

**Data :**

**$n_1 = 400$ rpm; $i = 4$; $z_1 = 15$; $\alpha = 20°$ FD; $m = 4$ mm; $b = 40$ mm**
**Pinion material – Forged steel**
**Gear material – High grade C.I.**

***Solution :***

$$i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1}$$

Speed of gear $n_2 = \frac{n_1}{i} = \frac{400}{4} = 100$ rpm

Number of teeth on gear $z_2 = iz_1 = 4 \times 15 = 60$

Pitch circle diameter of pinion $d_1 = mz_1 = 4 \times 15 = 60$

Pitch circle diameter of gear $d_2 = mz_2 = 4 \times 60 = 240$

From Table 23.18 (New DDHB)

Allowable static stress for forged steel untreated SAE 1040, $\sigma_{o1} = 173$ MPa

Allowable static stress for high grade CI, ASTM 50, $\sigma_{o2} = 103$ MPa

Lewis form factor for 20° full depth involute $y = 0.154 - \frac{0.912}{z}$ ---- 2.98(Old); 23.116 (New)

$$\therefore\ y_1 = 0.154 - \frac{0.912}{z_1} = 0.154 - \frac{0.912}{15} = 0.0932$$

$$y_2 = 0.154 - \frac{0.912}{z_2} = 0.154 - \frac{0.912}{60} = 0.1388$$

**Identify the weaker member**

| Particulars | $\sigma_o$ N/mm² | y | $\sigma_o y$ | Remarks |
|---|---|---|---|---|
| Pinion | 173 | 0.0932 | 16.1236 | |
| Gear | 103 | 0.1388 | 14.2464 | Weaker |

Since $\sigma_{o2}\, y_2 < \sigma_{o1}\, y_1$, gear is the weaker member. Therefore design should be based on gear.

Lewis equation for tangential tooth load $F_t = \sigma_o bypC_v$, or $\sigma_o bypK_v$ ---- 2.93(Old); 23.93 (New)

$$\sigma_o = \sigma_{02} = 103 \text{ N/mm}^2$$

$$y = y_2 = 0.1388$$

$$v_m = \frac{\pi d_2 n_2}{60000} = \frac{\pi \times 240 \times 100}{60000} = 1.257 \text{ m/sec}$$

$$K_v = \frac{3}{3 + v_m} \quad \text{since } v_m < 7.5 \text{ m/sec}$$ ---- 2.128(Old); 23.134a (New)

$$= \frac{3}{3 + 1.257} = 0.70478$$

$$p = \pi m = \pi \times 4 = 12.566 \text{ mm}$$

$\therefore$ Tangential tooth load on gear $F_{t2} = (103)(40)(0.1388)(12.566)(0.70478) = 5064.51$ N (Weaker member)

Also tangential tooth load $F_{t2} = 9550 \frac{NC_s}{n_2 r_2} = \frac{9550PC_s}{n_2 r_2}$ ---- 2.87b(Old DDHB); 23.87b (New DDHB)

where $n_2$ = Speed of gear; $r_2$ = pitch circle radius of gear in meters; N = Power in kW

$$r_2 = \frac{d_2}{2} = \frac{240}{2} = 120 \text{ mm} = 0.12 \text{ m};\ C_s = \text{Service factor} = 1$$

(Assume stready shock and 8 to 10 hrs duty per day, page 237 b)

$$\text{i.e., } 5064.51 = \frac{9550 \times P}{100 \times 0.12}$$

$\therefore$ Power transmitted P = N = 6.364 kW

---

*Example 4.4*

**Two spur gears are to be used for a rock crusher drive and are to be of minimum size. The gears are to be designed for the following requirements: Power to be transmitted is 18 kW, speed of pinion 1200 rev/min: velocity ratio 3.5 to 1, tooth profile 20° stub involute. Determine module and face width for strength requirements only.**

**Data :**

**$P = N = 18$ kW; $n_1 = 1200$ rpm; $i = 3.5$; $\alpha = 20°$ stub;**

***Solution :***

The diameter of the gears and the number of teeth on the gears are unknown. Referring Table 2.6 (Old DDHB); 23.6 (New DDHB), minimum number of teeth on pinion to avoid interference for 20° stub tooth is 14 $\therefore$ Select number of teeth on pinion $z_1 = 16$

$$i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1}$$

Number of teeth on gear $z_2 = iz_1 = 3.5 \times 16 = 56$

$$\text{Speed of gear } n_2 = \frac{n_1}{i} = \frac{1200}{3.5} = 342.857 \text{ rpm}$$

As the gear drive is to be of minimum or compact size, select strong materials for both pinion and gear. Select SAE 3245 ($C_r - N_i$ steel) for both pinion and gear. From Table 23.18 (New DDHB) for SAE 3245 ($C_r - N_i$ steel) $\sigma_{o1} = \sigma_{o2} = 418 - 517$ MPa select $\sigma_{o1} = \sigma_{o2} = 515$ MPa.

Lewis form factor for 20° stub $y = 0.17 - \dfrac{0.95}{z}$ ---- 2.99(Old DDHB); 23.117 (New DDHB)

$$\therefore \text{ Lewis form factor for pinion } y_1 = 0.17 - \frac{0.95}{z_1} = 0.17 - \frac{0.95}{16} = 0.110625$$

$$\text{Lewis form factor for gear } y_2 = 0.17 - \frac{0.95}{z_2} = 0.17 - \frac{0.95}{56} = 0.153036$$

**i) Identify the weaker member**

As pinion and gear material are the same, pinion is the weaker member. Therefore design should be based on pinion.

**ii) Design**

**a) Tangential tooth load on pinion** $F_{t1} = \dfrac{9550 \times 1000 N C_s}{n_1 r_1} = \dfrac{9550 \times 1000\, P C_s}{n_1 r_1}$

where $r_1$ = pitch circle radius of pinion in mm $= \dfrac{d_1}{2} = \dfrac{m z_1}{2} = \dfrac{m \times 16}{2} = 8\,m$

As the gears are used for rock crusher, assume intermittent with heavy shock.

$\therefore$ From Table 2.33 (Old DDHB); Table 23.13 (New DDHB) (Page 23.76) service factor $C_s = 1.5$

$$\text{i.e., } F_{t1} = \frac{9550\times 1000\times 18\times 15}{1200\times 8m} = \frac{17906.25}{m}\text{ N} \qquad \text{---- (i)}$$

**b) Lewis equation for tangential tooth load**

$F_t = \sigma_o\, b y\, p\, C_v = \sigma_o\, b y\, p\, K_v$ ---- 2.93(Old DDHB); 23.93 (New DDHB)

Since pinion is the weaker member

$F_{t1} = \sigma_{o1}\, b y_1\, p\, C_v = \sigma_{o1}\, b y_1\, p\, K_v$

$b = 3\,\pi m$ to $4\,\pi m$ or $9.5\text{ m} < b < 12.5\text{ m}$ ---- 2.126(Old DDHB); 23.132 (New DDHB)

$\therefore$ Select face width $b = 10\text{ m}$

$p$ = Circular pitch = $\pi m$

i.e., $F_{t1} = (515)(10\text{ m})(0.110625)(\pi m)(K_v) = 1789.824\text{ m}^2 C_v$ ---- (ii)

Equating equations (i) and (ii)

$$1789.824\,m^2 K_v = \frac{17906.25}{m}$$

$$\therefore\ \mathbf{m^3 K_v = 10} \qquad \text{---- (iii)}$$

Mean pitch line velocity $v_m = \dfrac{\pi d_1 n_1}{60000}$ ($\because$ Pinion weaker)

$$= \frac{\pi\times mz_1\times n_1}{60000} = \frac{\pi\times m\times 16\times 1200}{60000}$$

i.e., $\mathbf{v_m = 1.0053\,m}$ **---- m/sec**

**Trail : 1**

Select module m =2.5 mm [Select standard module from Table 2.3 (Old DDHB); Table 23.3 (New DDHB)]

$\therefore\ v_m = 1.0053\times 2.5 = 2.513$ m/sec.

Velocity factor $K_v = C_v = \dfrac{3}{3+v_m}$ since $v_m < 7.5$ m/sec---- 2.128(Old); 23.134a (New DDHB)

$$= \frac{3}{3+2.513} = 0.544$$

From equation (iii)

$\therefore\ (2.5)^3(0.544) \geq 10$

i.e., $8.5 < 10$

$\therefore$ Not suitable

**Trail : 2**

Select module $m = 3$mm

$v_m = 1.0053\times 3 = 3.016$ m/sec

$$\therefore K_v = C_v = \frac{3}{3+3.016} = 0.4987$$

From equation (iii)

$(3)^3 (0.4497) \geq 10$

i.e., $13.46 > 10$. Hence suitable

$\therefore$ Module $m = 3$ mm

**c) Check for the stress**

Allowable stress $\sigma_{all} = (\sigma_{o1} K_v)_{all} = 515 \times 0.4987 = 256.8305$ N/mm²

Induced stress $\sigma_{ind} = (\sigma_{o1} K_v)_{ind} = \dfrac{F_{t1}}{b y_1 p}$ ---- 2.93(Old DDHB); 23.93 (New DDHB)

$$= \frac{\left(\frac{17906.25}{3}\right)}{(10 \times 3)(0.110625)(\pi \times 3)} = 190.826 \text{ N/mm}^2$$

As $(\sigma_o K_v)_{ind} < (\sigma_o C_v)_{all}$, the design is safe.

$\therefore$ Module $m = 3$mm

Face width $b = 10\text{mm} = 10 \times 3 = 30$ mm

*Example 4.5*

**A pair of carefully cut spur gears with 20° full depth involute profile is used to transmit 12 kW at 1200 revolutions per minute of pinion. The gear has to rotate at 300 revolutions per minute. The material used for both pinion and gear is medium carbon steel whose allowable bending stress may be taken as 230 MPa. Determine the module and face width of the spur pinion and gear. Suggest suitable hardness. Take 24 teeth on pinion. Modulus of elasticity may be taken as 210 GPa** **VTU February 2002**

**Data :**

**$\alpha = 20°$ FD involute; $P = N = 12$ kW; $n_1 = 1200$ rpm; $n_2 = 300$ rpm; $\sigma_{o1} = \sigma_{o2} = 230$ MPa; $z_1 = 24$; $E_1 = E_2 = 210$ GPa $= 210 \times 10^3$ N/mm²; Carefully cut spur gear.**

***Solution :***

$$i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1}$$

$\therefore$ Velocity ratio $i = \dfrac{1200}{300} = 4$; $z_2 = iz_1 = 4 \times 24 = 96$

Lewis form factor for 20° FD involute $y = 0.154 - \dfrac{0.912}{z}$ ---- 2.98(Old); 23.116 (New DDHB)

$\therefore$ Lewis form factor for pinion $y_1 = 0.154 - \dfrac{0.912}{z_1} = 0.154 - \dfrac{0.912}{24} = 0.116$

Lewis form factor for gear $y_2 = 0.154 - \dfrac{0.912}{z_2} = 0.154 - \dfrac{0.912}{96} = 0.1445$

**i) Identify the weaker member**

As pinion and gear material are the same, pinion is the weaker member. Therefore design should be based on pinion.

**ii) Design**

**a) Tangential tooth load**

$$F_t = \frac{9550 \times 1000 NC_s}{nr} = \frac{9550 \times 1000\ PC_s}{nr}$$

where r in mm, N = P = Power in KW

$\therefore$ Tangential tooth load on the weaker member pinion $F_{t1} = \dfrac{9550 \times 1000 \times P \times C_s}{n_1 r_1}$

Assume medium shock and 8-10 hours duty per day

$\therefore$ From Table 2.33(Old DDHB); Table 23.13 (New DDHB) (Page 23.76) service factor $C_s = 1.5$

$$r_1 = \frac{d_1}{2} = \frac{mz_1}{2} = \frac{m \times 24}{2} = 12\ m$$

$$\text{i.e.,}\quad F_{t1} = \frac{9550 \times 1000 \times 12 \times 1.5}{1200 \times 12m} = \frac{11937.5}{m} \qquad \text{---- (i)}$$

**b) Lewis equation for tangential tooth load $F_t = \sigma_o\, by\, p\, C_v = \sigma_o\, by\, p\, K_v$**

---- **2.93**(Old); 23.93 (New DDHB)

Since pinion is the weaker member

$F_{t1} = \sigma_{o1}\, by_1\, p\, C_v = \sigma_{o1}\, by_1\, p\, K_v$

Face width $b = 3\pi m$ to $4\pi m$ or $9.5\, m < b < 12.5\, m$ ---- 2.126 (Old DDHB)

$\therefore$ Select face width $b = 10\, m$; circular pitch $p = \pi m$ 23.132 (New DDHB)

i.e. $F_{t1} = (230)(10\, m)(0.116)(\pi m)(K_v) = 838.177\, m^2\, K_v$ ---- (ii)

Equating equations (i) and (ii)

$$838.177\, m^3 K_v = \frac{11937.5}{m}$$

$$\therefore\ \mathbf{m^3 K_v = 14.242} \qquad \text{---- (iii)}$$

Mean pitch line velocity of the weaker member pinion $v_m = \dfrac{\pi d_1 n_1}{60000}$

$$= \frac{\pi m z_1 n_1}{60000} = \frac{\pi \times m \times 24 \times 1200}{60000} = 1.508m\ \text{m/sec}$$

**Trail : 1**

Select module $m = 3$ mm [Select standard module from Table 2.3(Old DDHB); Table 23.3 (New DDHB)]

$\therefore\ v_m = 1.508 \times 3 = 4.524$ m/sec

$$K_v = C_v = \frac{3}{3 + v_m} = \frac{3}{3 + 4.524} = 0.3987 \quad \text{Since } v_m < 7.5 \text{ m/sec}$$

---- 2.128(Old DDHB); 23.134a (New DDHB)

Now from equation (iii)

$(3)^3\,(0.3987) \geq 14.242$

$10.765 < 14.242$

$\therefore$ Not suitable

**Trail : 2**

Select module $m = 4$ mm; $v_m = 1.508 \times 4 = 6.032$ m/sec

$$K_v = C_v = \frac{3}{3+6.032} = 0.332$$

From equation (iii)

$(4)^3(0.332) \geq 14.242$; $21.248 > 14.242$. Hence suitable

$\therefore$ Module $m = 4$mm

**c) Check for the stress**

Allowable stress $\sigma_{all} = (\sigma_{ol} K_v)_{all} = 230 \times 0.332 = 76.36$ N/mm²

Induced stress $\sigma_{ind} = (\sigma_{ol} K_v)_{ind} = \frac{F_{t1}}{by_1 p}$ ---- 2.93(Old DDHB); 23.93 (New DDHB)

$$= \frac{\left(\frac{11937.5}{4}\right)}{(10\times4)(0.116)(\pi\times4)} = 51.183 \text{ N/mm}^2$$

Since $(\sigma_{ol} K_v)_{ind} < (\sigma_{ol} K_v)_{all}$, the design is safe.

Also in order to avoid the breakage of gear tooth due to bending, the beam strength should be more than the effective force between the meshing teeth

Effective force $F_{eff} = \frac{F_{t1} C_s}{K_v} = \frac{F_{t1}}{K_v}$ since $C_s$ is already considered

$$= \frac{\left(\frac{11937.5}{4}\right)}{0.332} = 8989.1 \text{ N}$$

Beam strength of weaker member $F_{b1} = \sigma_{b1}\, b y_1 p = (230)(10\times4)(0.116)(\pi\times4) = 13410.83$ N

$F_{b1} > F_{eff}$

$$\therefore \text{FOS} = \frac{F_{b1}}{F_{eff}} = \frac{13410.83}{8989.1} = 1.5$$

The design is satisfactory and hence module should be equal to 4 mm.

$\therefore$ Module $m = 4$mm

Face width $b = 10m = 10\times4 = 40$ mm

**iii) Dynamic load**

**a) Dynamic load** $F_d = F_t + \frac{21 v_m (F_t + bC)}{21 v_m + \sqrt{F_t + bC}}$ ---- 2.148 a(Old DDHB); 23.155 (New DDHB)

**From Fig. 2.29 (Old DDHB); Fig. 23.34a (New DDHB) for carefully cut gears (Class-II gears) for m = 4 mm**

Error $f = 0.025$ mm

From Table 2.35 (Old DDHB); Table 23.32 (New DDHB) for f = 0.025 mm, $\alpha = 20°$ full depth and Steel pinion – Steel gear

Dynamic load factor $C = 290\frac{kN}{m} = 290$ N/mm

Tangential tooth load $F_t = F_{t1} = \frac{11937.5}{m} = \frac{11937.5}{4} = 2984.375$ N

$$\text{i.e., } F_d = 2984.375 + \frac{21 \times 6.032[2984.375 + 40 \times 290]}{21 \times 6.032 + \sqrt{2984.375 + 40 \times 290}}$$

$\therefore$ Dynamic load $F_d = 10450.6$ N

**b) Wear load $F_w = d_1 bQK$** ---- 2.160 (Old DDHB); 23.160 (New DDHB)

Ratio factor $Q = \frac{2z_2}{z_1 + z_2} = \frac{2 \times 96}{24 + 96} = 1.6$---- 2.163 (Old DDHB); 23.163 (New DDHB)

Equivalent Young's Modulus $E_o = \frac{2E_1E_2}{E_1 + E_2}$ ---- 2.162(Old DDHB); 23.162 (New DDHB)

$$= \frac{2 \times 210 \times 10^3 \times 210 \times 10^3}{210 \times 10^3 + 210 \times 10^3} = 210 \times 10^3 \text{ N/mm}^2$$

Load stress factor $K = \frac{1.43\sigma_{-1c}^2 \sin\alpha}{E_o} = \frac{1.43\sigma_{fac}^2 \sin\alpha}{E_o}$ ---- 2.161(Old DDHB); 23.161 (New DDHB)

Pitch circle diameter of pinion $d_1 = mz_1 = 4 \times 24 = 96$

**For safer design**

$$F_w \geq F_d$$

i.e., $d_1 bQK \geq 10450.6$

i.e., $(96)(40)(1.6)K \geq 10450.6$

$\therefore$ Load stress factor $K \geq 1.701$ N/mm²

$$\text{i.e., } \frac{1.43\sigma_{faC}^2 \sin\alpha}{E_o} \geq 1.701$$

$$\text{i.e., } \frac{1.43\left(\sigma_{fac}^2\right)^2 \sin 20}{210 \times 10^3} \geq 1.701$$

$\therefore$ $\sigma_{fac} \geq 854.6$ N/mm²

From Table 2.40 (Old DDHB); Table 23.37B (New DDHB) for $\alpha = 20°$ FD, $K \geq 1.701$ N/mm² and $\sigma_{fac} \geq 854.6$ N/mm²

Surface hardness for pinion = 400 BHN

Surface hardness for gear = 300 BHN

*Example 4.6*

**Design a pair of spur gears to transmit 20 kW from a shaft rotating at 1000 rpm to a parallel shaft which is to rotate at 310 rpm. Assume number of teeth on pinion 31 and 20° full depth tooth form. The material for pinion is C 45 steel untreated and for gear cast steel 0.20% C untreated.**

**VTU August 2001**

**Data :**

**$P = N = 20$ kW; $n_1 = 1000$ rpm; $n_2 = 310$ rpm; $z_1 = 31$; $\alpha = 20°$ FD; pinion material – C 40 steel untreated; Gear material – Cast steel 0.20% C untreated**

*Solution :*

$$i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1}$$

$$\therefore \text{ Velocity ratio } i = \frac{n_1}{n_2} = \frac{1000}{310} = 3.2258$$

Number of teeth on gear $z_2 = iz_1 = 3.2258 \times 31 = 100$

Lewis form factor for 20° FD involute $y = 0.154 - \frac{0.912}{z}$ ---- 2.98(Old); 23.116 (New DDHB)

Lewis form factor for pinion $y_1 = 0.154 - \frac{0.912}{z_1} = 0.154 - \frac{0.912}{31} = 0.12458$

Lewis form factor for gear $y_2 = 0.154 - \frac{0.912}{z_2} = 0.154 - \frac{0.912}{100} = 0.14488$

From Table 23.18 (New DDHB) Allowable stress for

C 45 steel untreated (i.e., steel SAE 1045 untreated) $\sigma_{01} = 207$ MPa

Cast steel 0.20% c untreated $\sigma_{02} = 138$ MPa

**i) Identify the weaker member**

| Particulars | $\sigma_o$ N/mm² | y | $\sigma_o y$ | Remarks |
|---|---|---|---|---|
| Pinion | 207 | 0.12458 | 25.788 | |
| Gear | 138 | 0.14488 | 19.9934 | Weaker |

As $\sigma_{02} y_2 < \sigma_{01} y_1$, Gear is weaker. Therefore design should be based on gear.

**ii) Design**

**a) Tangential tooth load**

$$F_t = \frac{9550 \times 1000 \times NC_s}{nr} = \frac{9550 \times 1000 \times PC_s}{nr} \quad \text{where r in mm}$$

$$\therefore \text{ Tangential tooth of the weaker member gear } F_{t_2} = \frac{9550 \times 1000 \times NC_s}{n_2 r_2} = \frac{9550 \times 1000 \times PC_s}{n_2 r_2}$$

Pitch circle radius of gear $r_2 = \frac{d_2}{2} = \frac{mz_2}{2} = \frac{m \times 100}{2} = 50\text{ m}$

Assume medium shock and 8 – 10 hrs duty per day

∴ From Table 2.33 (Old DDHB); Table 23.13 (New DDHB), (Page 23.76), service factor $C_s = 1.5$

$$\text{i.e. } F_{t_2} = \frac{9550 \times 1000 \times 20 \times 1.5}{310 \times 50m} = \frac{18483.871}{m} \quad \text{---- (i)}$$

**b) Lewis equation for tangential tooth load $F_t = \sigma_o\, by\, p\, C_v$**---- 2.93(Old); 23.93 (New DDHB)

As gear is the weaker member

$$F_{t_2} = \sigma_{02}\, by_2\, p\, C_v = \sigma_{02}\, by_2\, p\, K_v$$

Face width $b = 3\pi m$ to $4\pi m$ or $9.5\text{ m} < b < 12.5\text{ m}$ ---- 2.126(Old); 23.132 (New DDHB)

Select $b = 10\text{ m}$; Circular pitch $p = \pi m$

$$\text{i.e. } F_{t_2} = (138)(10\text{ m})(0.14488)(\pi m) K_v = 628.11\, m^2 K_v \quad \text{---- (ii)}$$

Equating equations (i) and (ii)

$$628.11\, m^2 K_v = \frac{18483.871}{m}$$

$$\therefore\ m^3 K_v = 29.428 \quad \text{---- (iii)}$$

Mean pitch line velocity of the weaker member gear $v_m = \frac{\pi d_2 n_2}{60000}$

$$= \frac{\pi m z_2 n_2}{60000} = \frac{\pi \times m \times 100 \times 310}{60000} = 1.623m,\ \text{m/sec}$$

**Trail : 1**

Select module $m = 4\text{ mm}$ [Select standard module from Table 2.3 (Old); Table 23.3 (New DDHB)]

$\therefore\ v_m = 1.623 \times 4 = 6.5$ m/sec

Velocity factor $K_v = C_v = \frac{3}{3 + v_m} = \frac{3}{3 + 6.5} = 0.316$ since $v_m < 7.5$ m/sec ---- 2.128(Old); 23.134a (New)

Now from equation (iii)

$(4)^3 (0.316) \geq 29.428$

$20.224 < 29.428$

∴ Not suitable

**Trail : 2**

Select module $m = 5\text{mm}$

$\therefore\ v_m = 1.623 \times 5 = 8.115$ m/sec

Velocity factor $K_v = C_v = \frac{4.5}{4.5 + v_m} = \frac{4.5}{4.5 + 8.115} = 0.35672$ ($\because v_m < 12.5$ m/sec.)

---- 2.129(Old DDHB); 23.135a (New DDHB)

Now from equation (iii)

$(5)^3 (0.35672) \geq 29.428$; $44.59 > 29.428$. Hence suitable

∴ Module $m = 5$ mm

**c) Check for the stress**

Allowable stress $\sigma_{all} = (\sigma_{o2} K_v)_{all} = (138)(0.35672) = 49.227$ N/mm²

Induced stress $\sigma_{ind} = (\sigma_{o2} K_v)_{ind} = \dfrac{F_{t2}}{by_2p}$ ---- 2.93 (Old DDHB); 23.93 (New DDHB)

$$= \frac{\left(\frac{18483.871}{5}\right)}{(10\times5)(0.14488)(\pi\times5)} = 32.488 \text{ N/mm}^2$$

Since $(\sigma_{o2} K_v)_{ind} < (\sigma_{o2} K_v)_{all}$, the design is safe. Also in order to avoid the breakage of gear tooth due to bending, the beam strength should be more than the effective force between the meshing teeth

Effective force $F_{eff} = \dfrac{F_{t_2}.C_s}{K_v} = \dfrac{F_{t_2}}{K_v}$ since $C_s$ is already considered

$$= \frac{\left(\frac{18483.871}{5}\right)}{0.35672} = 10363.24 \text{ N}$$

Beam strength of weaker member $F_{b2} = \sigma_{o2}\, by_2p = (138)(10\times5)(0.14488)(\pi\times5) = 15702.81$ N

$$F_{b2} > F_{eff}$$

$$\therefore \text{FOS} = \frac{F_{b2}}{F_{eff}} = \frac{15702.81}{10363.24} = 1.515$$

The design is satisfactory and hence the module should be equal to 5 mm

$\therefore$ Module m = 5mm

**iii) Dimensions**

Module m = 5mm

**From Table 2.1** (Old DDHB); Table 23.1 (New DDHB) for 20° full depth involute system

Addendum $h_a = 1\,m = 1\times5 = 5$ mm

Dedendum $h_f = 1.25\,m = 1.25\times5 = 6.25$ mm

Working depth $h' = 2\,m = 2\times5 = 10$ mm

Total depth $h = 2.25\,m = 2.25\times5 = 11.25$ mm

Tooth thickness $s = \dfrac{\pi}{2}m = \dfrac{\pi}{2}\times5 = 7.854$ mm

Minimum clearance $c = 0.25\,m = 0.25\times5 = 1.25$ mm

Pitch diameter of pinion $d_1 = z_1m = 31\times5 = 155$ mm

Pitch diameter of gear $d_2 = z_2m = 100\times5 = 500$ mm

Outside or Addendum diameter of pinion $d_{a1} = (z_1+2)\,m = (31+2)\,5 = 165$ mm

Outside or Addendum diameter of gear $da_2 = (z_2+2)\,m = (100+2)\,5 = 510$ mm

Centre distance $a = \dfrac{m}{2}(z_1+z_2) = \dfrac{5}{2}(31+100) = 327.5$ mm---- 2.19(Old); 23.19 (New)

Dedendum or Root circle diameter of pinion $d_{f1} = d_1 - 2h_f = 155 - 2\times6.25 = 142.5$ mm

---- 2.16(Old DDHB); 23.16 (New DDHB)

Dedendum circle diameter of gear $d_{f2} = d_2 - 2h_f = 500 - 2\times6.25 = 487.5$ mm

Base circle diameter of pinion $d_{b1} = d_1 \cos\alpha = 155 \cos 20 = 145.652$ ---- 2.9(Old); 23.9 (New)

Base circle diameter of gear $d_{b2} = d_2 \cos\alpha = 500 \cos 20 = 469.846$

Tangential tooth load $F_t = \frac{18483.871}{m} = \frac{18483.871}{5} = 3696.774$ N ---- from (i)

Mean pitch line velocity $v_m = 8.115$ m/sec

Face width $b = 10\,m = 10 \times 5 = 50$ mm.

Circular pitch $p = \pi m = \pi \times 5 = 15.708$ mm

Velocity factor $K_v = C_v = 0.35672$; service factor $C_s = 1.5$

**iv) Checking**

**a) Dynamic load**

$$\text{Dynamic load } F_d = F_t + \frac{21 v_m (F_t + bC)}{21 v_m + \sqrt{F_t + bC}}$$

**From Fig. 2.30 (Old DDHB); Fig. 23.35a (New DDHB) for $v_m = 8.115$ m/sec**

Error f = 0.045 mm

**From Table 2.35 (Old DDHB); Table 23.32 (New DDHB)** for steel pinion-steel gear and 20° full depth

For error f = 0.025 mm, $C = 290 \frac{KN}{m} = 290$ N/mm

For error f = 0.05 mm, $C = 580 \frac{KN}{m} = 580$ N/mm

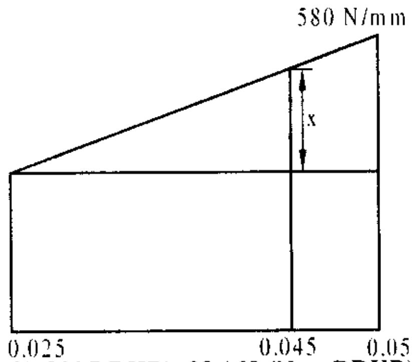
**By interpolation**

$$\frac{x}{580 - 290} = \frac{0.045 - 0.025}{0.05 - 0.025}$$

$\therefore\ x = 232$ N/mm

$\therefore$ For error f = 0.045 mm;

Load factor C = 290 + 232 = 522 N




$$\text{i.e., } F_d = 3696.774 + \frac{21 \times 8.115[3696.774 + 50 \times 522]}{21 \times 8.115 + \sqrt{3696.774 + 50 \times 522}}$$

$\therefore$ Dynamic load $F_d = 18499.507$ N

**b) Wear load**

Wear load $F_w = d_1 bQK$ ---- 2.160(Old DDHB); 23.160 (New DDHB)

Ratio factor $Q = \frac{2z_2}{z_1 + z_2} = \frac{2 \times 100}{31 + 100} = 1.52672$

**For safer design** $F_w \geq F_d$

i.e., $d_1 bQK \geq F_d$

i.e., (155) (50) (1.52672) (K) ≥ 18508.11

$\therefore$ Load stress factor $K \geq 1.5642$ N/mm²

From Table 2.40 (Old DDHB) ; Table 23.37 B (New DDHB) for $\alpha = 20°$ FD and $K \geq 1.5642$ N/mm²

Surface hardness for pinion = 350 BHN

Surface hardness for gear = 300 BHN

***Example 4.7***

**Design a bronze spur gear 81.4 MN/m² and mild steel pinion 101 MN/m² to transmit 5 kW at 1800 rpm. The velocity ratio is 3.5:1. Pressure angle is 14½°. Not less than 15 teeth are to be used on either gear. Determine the module and face width. Also, suggest suitable surface hardness for the weaker member based on dynamic and wear considerations.**

**Data :**

$\sigma_{01} = 101$ **MN/m² = 101 N/mm²;** $\sigma_{02} = 81.4$ **MN/m² = 81.4 N/mm²; P = N = 5 kW;** $n_1 = 1800$ **rpm;**
$i = 3.5$; $\alpha = 14\frac{1}{2}°$; $z_1 \geq 15$

***Solution :***

$$\text{Velocity ratio } i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1}$$

$$\therefore \text{ Speed of gear } n_2 = \frac{n_1}{i} = \frac{1800}{3.5} = 514.285 \text{ rpm}$$

Select number of teeth on pinion $z_1 = 16$

$\therefore$ Number of teeth on gear $z_2 = iz_1 = 3.5 \times 16 = 56$

Lewis form factor for 14½° involute $y = 0.124 - \frac{0.684}{z}$ ---- 2.97(Old DDHB); 23.115 (New DDHB)

$$\text{Lewis form factor for pinion } y_1 = 0.124 - \frac{0.684}{z_1} = 0.124 - \frac{0.684}{16} = 0.08125$$

$$\text{Lewis form factor for gear } y_2 = 0.124 - \frac{0.684}{z_2} = 0.124 - \frac{0.684}{56} = 0.11179$$

**i) Identify the weaker member**

| Particulars | $\sigma_o$ N/mm² | y | $\sigma_o y$ | Remarks |
|---|---|---|---|---|
| Pinion | 101 | 0.08125 | 8.206 | Weaker |
| Gear | 81.4 | 0.11179 | 9.1 | |

Since $\sigma_{01}\, y_1 < \sigma_{02}\, y_2$, pinion is weaker. Therefore design should be based on pinion.

**ii) Design**

**a) Tangential tooth load** $F_t = \frac{9550 \times 1000 \times NC_s}{nr} = \frac{9550 \times 1000 \times PC_s}{nr}$ **where r in mm**

$\therefore$ Tangential tooth load of the weaker member $F_{t_1} = \frac{9550 \times 1000 \times PC_s}{n_1 r_1}$

Pitch circle radius of pinion $r_1 = \frac{d_1}{2} = \frac{mz_1}{2} = \frac{m \times 16}{2} = 8\text{ m}$

Assume medium shock and 8 – 10 hrs duty per day

∴ From Table 2.33 (Old DDHB); Table 23.13 (New DDHB) (Page 23.76), service factor $C_s = 1.5$

$$\text{i.e., } F_{t_1} = \frac{9550 \times 1000 \times 5 \times 1.5}{1800 \times 8m} = \frac{4973.96}{m} \quad \text{---- (i)}$$

**b) Lewis equation for tangential tooth load** $F_t = \sigma_o \, by \, pC_v = \sigma_o \, by \, pK_v$

**---- 2.93(Old DDHB); 23.93 (New DDHB)**

∴ Tangential tooth load on weaker member $F_{t1} = \sigma_{01} \, by_1 \, pC_v$

face width $b = 3\,\pi m$ to $4\,\pi m$ or $9.5\text{ m} < b < 12.5\text{ m}$ ---- 2.126(Old); 23.132 (New DDHB)

Select $b = 10\text{ m}$; circular pitch $p = \pi m$

$$\text{i.e. } F_{t_1} = (101)(10\text{ m})(0.08125)(\pi m)\,K_v = 257.807\text{ m}^2\,K_v \quad \text{---- (ii)}$$

Mean pitch line velocity of weaker member $v_m = \dfrac{\pi d_1 n_1}{60000} = \dfrac{\pi m z_1 n_1}{60000} = \dfrac{\pi \times m \times 16 \times 1800}{60000} = 1.508\text{ mm/sec}$

Equating equations (i) and (ii)

$$257.807\,m^2 K_v = \frac{4973.96}{m}$$

$$\therefore \; m^3 K_v = 19.293 \quad \text{---- (iii)}$$

**Trail : 1**

Select module m = 3 mm [Select standard module from Table 2.3 (Old); Table 23.3 (New DDHB)]

$$\therefore \; v_m = 1.508 \times 3 = 4.524\text{ m/sec}$$

Velocity factor $K_v = C_v = \dfrac{3}{3 + v_m} = \dfrac{3}{3 + 4.524} = 0.4$. Since $v_m < 7.5$ m/sec

---- 2.128(Old DDHB); 23.134a (New DDHB)

From equation (iii)

$$(3)^3(0.4) \geq 19.293$$

$$10.766 < 19.293$$

∴ Not suitable

**Trail : 2**

Select module $m = 4\text{mm}$

$$v_m = 1.508 \times 4 = 6.032\text{ m/sec}$$

$$K_v = C_v = \frac{3}{3 + 6.032} = 0.3322$$

From equation (iii)

$$(4)^3(0.3322) \geq 19.293$$

i.e., $21.258 > 19.293$ Hence suitable

∴ Module $m = 4$ mm

**c) Check for the stress**

Allowable stress $\sigma_{all} = (\sigma_{01} K_v)_{all} = (101)(0.3322) = 33.552\text{ N/mm}^2$

Induced stress $\sigma_{ind} = (\sigma_{01} K_v)_{ind} = \dfrac{F_{t_1}}{by_1 p}$ ---- 2.93(Old DDHB); 23.93 (New DDHB)

$$= \frac{\left(\frac{4973.96}{4}\right)}{(10\times4)(0.08125)(\pi\times4)} = 30.447\ \text{N/mm}^2$$

Since $(\sigma_{01} K_v)_{ind} < (\sigma_{01} K_v)_{all}$, the design is safe.

Also in order to avoid the breakage of gear tooth due to bending, the beam strength should be more than the effective force between the meshing teeth.

$$\text{Effective force } F_{eff} = \frac{F_{t_1}.C_s}{K_v} = \frac{F_{t_1}}{K_v} \text{ since } C_s \text{ is already considered} = \frac{\left(\frac{4973.96}{4}\right)}{0.3322} = 3743.2\ \text{N}$$

Beam strength of weaker member $F_{b_1} = \sigma_{01}\, b\, y_1\, p$

$$= (101)(10\times4)(0.08125)(\pi\times4) = 4124.91\ \text{N}$$

$$F_{b_1} > F_{eff}$$

$$\therefore\ \text{FOS} = \frac{Fb_1}{F_{eff}} = \frac{4124.9}{3743.2} = 1.1$$

The design is satisfactory and hence the module should be equal to 4 mm

$\therefore$ Module $m = 4\text{mm}$

Face width $b = 10m = 10\times4 = 40\text{mm}$.

**iii) Checking**

**a) Dynamic load** $F_d = F_t + \dfrac{21v_m(F_t + bC)}{21v_m + \sqrt{F_t + bC}}$ **---- 2.148 a(Old DDHB); 23.155 (New DDHB)**

$\therefore$ Tangential tooth load $F_t = \dfrac{4973.96}{m} = \dfrac{4973.96}{4} = 1243.5\ \text{N}$ ---- from (i)

Mean pitch line velocity $v_m = 6.032$ m/sec

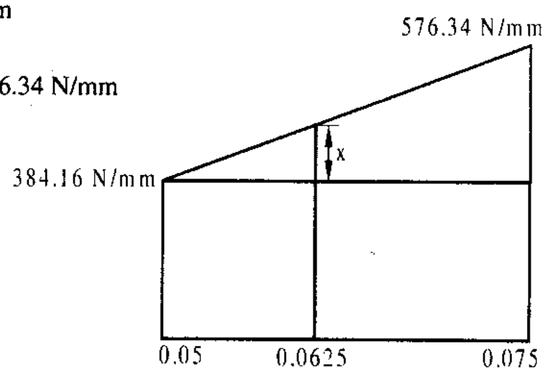
**From Table 2.30; (Old DDHB); Fig. 23.35a (New DDHB), for $v_m = 6.032$ m/sec**

Error f = 0.0625 mm

**From Table 2.35 (Old DDHB); Table 23.32 (New DDHB)** for $\alpha = 14\frac{1}{2}°$, taking steel – CI combination

For $f = 0.05$ mm ; $C = 384.16 \dfrac{KN}{m} = 384.16$ N/mm

For $f = 0.075$ mm ; $C = 576.34 \dfrac{KN}{m} = 576.34$ N/mm




**By interpolation**

$$\frac{x}{576.34 - 384.16} = \frac{0.0625 - 0.05}{0.075 - 0.05}$$

$$\therefore\ x = 96.09\ \text{N/mm}$$

$\therefore$ For error $f = 0.0625$ mm;

Dynamic factor $C = 384.16 + 96.09 = 480.25$ N/mm

$$\text{i.e.,}\quad F_d = 1243.5 + \frac{21\times 6.032[1243.5+40\times 480.25]}{21\times 6.032+\sqrt{1243.5+40\times 480.25}} = 10850.484 \text{ N}$$

**b) Wear load**

Wear load $F_w = d_1 bQK$ ---- 2.160(Old DDHB); 23.160 (New DDHB)

$$\text{Ratio factor } Q = \frac{2z_2}{z_1+z_2} = \frac{2\times 56}{16+56} = 1.556$$

Pitch circle diameter of pinion $d_1 = mz_1 = 4\times 16 = 64$

From Table 2.8 [Old DDHB - Volume-I] or Table 2.10 (New DDHB - Volume-I)

For carbon steel pinion, Young's modulus $E_1 = 206$ GPa $= 206\times 10^3$ N/mm²

For bronze gear, Young's modulus $E_2 = 96$ GPa $= 96\times 10^3$ N/mm²

$$\therefore \text{ Equivalent Young's modulus } E_o = \frac{2E_1E_2}{E_1+E_2} = \frac{2\times 206\times 10^3\times 96\times 10^3}{206\times 10^3+96\times 10^3} = 130.967\times 10^3 \text{ N/mm}^2$$

$$K = \frac{1.43\sigma_{-1C}^2 \sin\alpha}{E_o} = \frac{1.43\sigma_{fac}^2 \sin\alpha}{E_o}$$ ---- 2.161 (Old DDHB); 23.161 (New DDHB)

**For safer design** $F_w \geq F_d$

i.e., $d_1 bQK \geq F_d$

i.e., $(64)(40)(1.556)(K) \geq 10850.484$

$\therefore$ Load stress factor $K \geq 2.724$

$$\text{i.e.,}\quad \frac{1.43\sigma_{fac}^2 \times \sin\alpha}{E_o} \geq 2.724$$

$$\text{i.e.,}\quad \frac{1.43\times\sigma_{fac}^2\times\sin 14.5}{130.967\times 10^3} \geq 2.724$$

$\therefore$ Limiting stress for surface fatigue $\sigma_{fac} \geq 998.2$ N/mm²

From Table 23.37B New (DDHB) for $\sigma_{fac} = 998.2$ N/mm²

Surface hardness of pinion = 400 BHN

Surface hardness of gear = 400 BHN

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*Example 4.8 :*

**In an automobile gear box, the second speed gear shaft is to be driven from main shaft with velocity ratio 1.5:1. Main shaft transmits 12 kW at 3000 rpm. The shaft centre distance = 80 mm Pinion material is cast steel heat treated and gear material is cast steel untreated. Profile of the gear is 20° stub involute. Determine (i) Module, (ii) Face width (iii) Number of teeth on pinion and gear.**

**Data :**

**$i = 1.5$; $P = N = 12$ kW; $n_1 = 3000$ rpm; $a = 80$ mm; $\alpha = 20°$ stub involute**

**Pinion material – Cast steel heat treated. Gear material – Cast steel untreated.**

*Solution :*

$$\text{Velocity ratio } i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1}$$

$\therefore$ Speed of gear $n_2 = \frac{n_1}{i} = \frac{3000}{1.5} = 2000$ rpm

Centre distance $a = \frac{m(z_1+z_2)}{2} = \frac{d_1+d_2}{2} = \frac{d_1+id_1}{2} = \frac{d_1(1+i)}{2}$ ----2.19(Old); 23.19 (New DDHB)

i.e., $80 = \frac{d_1(1+1.5)}{2}$

$\therefore$ Pitch circle diameter of pinion $d_1 = 64$ mm

Pitch circle diameter of gear $d_2 = id_1 = 1.5 \times 64 = 96$ mm

From Table 23.18 (New DDHB)

For cast steel heat treated $\sigma_{01} = 173$ MPa $= 173$ N/mm$^2$

For cast steel untreated $\sigma_{02} = 138$ MPa $= 138$ N/mm$^2$

To identify weaker member temporarily assume $z_1 = 20$

$\therefore z_2 = iz_1 = 1.5 \times 20 = 30$

Lewis form factor for 20° stub involute $y = 0.17 - \frac{0.95}{z}$ ----2.99(Old DDHB); 23.117 (New DDHB)

$\therefore$ Lewis form factor for pinion $y_1 = 0.17 - \frac{0.95}{z_1} = 0.17 - \frac{0.95}{20} = 0.1225$

Lewis form factor for gear $y_2 = 0.17 - \frac{0.95}{z_2} = 0.17 - \frac{0.95}{30} = 0.1383$

**i) Identify the weaker member**

| Particulars | $\sigma_o$ N/mm$^2$ | y | $\sigma_o y$ | Remarks |
|---|---|---|---|---|
| Pinion | 173 | 0.1225 | 21.1925 | |
| Gear | 138 | 0.1383 | 19.0854 | Weaker |

Since $\sigma_{02} y_2 < \sigma_{01} y_1$, gear is weaker member. Therefore design should be based on gear.

**ii) Design**

**a) Tangential tooth load** $F_t = \frac{9550 \times 1000 \times NC_s}{nr} = \frac{9550 \times 1000 \times PC_s}{nr}$ **where r in mm**

$\therefore$ Tangential tooth load of the weaker member $F_{t_2} = \frac{9550 \times 1000 \times NC_s}{n_2 r_2} = \frac{9550 \times 1000 \times PC_s}{n_2 r_2}$

Pitch circle radius of gear $r_2 = \frac{d_2}{2} = \frac{96}{2} = 48$

Assume medium shock and 8 – 10 hours duty per day

$\therefore$ From Table 2.33(Old DDHB); Table 23.13 (New DDHB) (Page 23.76); service factor $C_s = 1.5$